Payment Rules for Unit Commitment Dispatch

In the current design of U.S. electricity spot markets, the generation dispatch mechanism and the payment rule are incompatible with each other. The uniform-price auction format predicated on two-sided market design with marginal pricing is flawed, since the supply and the demand are not treated equitably and discrete decisions, such as unit commitment, are inevitable. The pay-as-bid scheme is better suited for the market reality.

Yanchao Liu and Michael C. Ferris

I. Introduction

A substantial portion\(^1\) of the electricity trades in the U.S. wholesale energy markets go through the spot markets organized by independent system operators (ISO) or regional transmission organizations (RTO).\(^2\) Essentially, ISOs need to organize two things: the electricity flow and the cash flow. For the former, ISOs schedule and dispatch the supply and demand in an economic and reliable way, and for the latter, they make and enforce payment rules to fairly allocate the costs and revenues among market participants. These tasks turn out to be problematic, given the complex nature of the commodity.

Electricity differs from an ordinary commodity in two ways: (1) what is produced now cannot be practically stored for use later, as the production rate and capacity exceed the storage rate and capacity by orders of magnitude; (2) the demand for electricity exhibits huge, although to some extent predictable,
variations throughout a day, the amplitude of which is way greater than the capacity of any individual generating unit. As a result, many generating units are forced to run intermittently, with frequent startups and shutdowns, as well as prolonged periods of no-load “idling.” As in other industrial processes, startup and idling of a unit incur extra costs, but unlike other industries, these costs have significant effect on the overall cost of electricity production due to their frequency and must be compensated for in the settlement.

Such characteristics of the electricity supply are recognized in the ISOs’ market operations. The discrete decisions, i.e., whether to start and shut down a generator, have been an integral part in ISOs’ generator dispatch algorithms, and a compensation rule for the discrete activities is also included in most tariffs. However, the ways these discrete decisions or activities are treated in the dispatch algorithm and in the payment rule are disjoint and incompatible, which undermines the overall fairness and efficiency. This article reviews the issues with the current payment rule, justifies an alternative pay-as-bid scheme, and presents some preliminary results regarding bidders’ response under pay-as-bid.

II. The Existing Payment Rule Is Problematic

In the day-ahead markets, ISOs take bids from the generators, and make the unit commitment and dispatch decisions by solving the security-constrained unit commitment and economic dispatch model (UCED) based on the bid data. The UCED model minimizes the total generation cost, subject to the supply-demand balance constraints, transmission network constraints, and the generators’ operational constraints as specified in the bids. The operational constraints include minimum up-time and down-time constraints, ramp-up and ramp-down constraints, and lower and upper bounds on the output level once committed. This model is a mixed integer program (MIP) that can be efficiently solved using modern optimization technology. Given a particular commitment decision, e.g., the one arising from the solution of UCED, the economic dispatch (ED) model, which is a linear program (LP), finds the optimal dispatch that minimizes the total energy cost. The optimal multiplier value, or the shadow price, of the power balance constraint in the ED model represents the cost of satisfying the next increment of demand, and is set as the market clearing price (MCP), or locational marginal price (LMP) to emphasize its “locational” dimension. In fact, MCP is both locational and temporal, indicating the price at a particular location during a particular time period. This interpretation is implied when we use the term “MCP” in the remainder of the article.

As for the payment, all committed generators are paid for the MWh energy output at the uniform MCP. However, as pointed out by Johnson et al., “the commitment which predicates the optimal dispatch phase strongly affects the market clearing prices … these prices are suboptimal since they do not reflect the inter-temporal costs and constraints that drive the unit commitment.” It has been acknowledged that in the context of unit commitment, there may not be a uniform price (i.e., MCP) that supports the efficient market equilibrium. In other words, the socially optimal dispatch solution cannot be achieved by the market participants’ profit-maximizing response to a uniform price. To enforce the central dispatch solution, ISOs have to grant generators “make-whole” payments to compensate their opportunity loss, as well as the unit commitment costs, incurred by complying with the central dispatch and giving up their profit-maximization in response.
to the MCP. Such payments contribute to the ISOs' overall procurement cost and eventually show up on consumers' electricity bills as "uplift" costs.

"Make-whole" payments are addressed by different names within different ISOs. For example, ISO New England uses the term "Net Commitment Period Compensation," Midwest ISO uses "Offer Revenue Sufficiency Guarantee Payment," PJM includes them in the "Operating Reserves Credit," CAISO uses "Bid Cost Recovery (BCR) uplift payment," and New York ISO uses "Bid Production Cost Guarantee Payments." Despite the differing names and formula, generally speaking, a make-whole payment to a generator is calculated as the positive difference between the generator's as-bid cost (including energy, no-load, and start-up) and its energy revenue paid at the MCP, evaluated at the actual commitment and dispatch solution. If the energy revenue at MCP exceeds the as-bid cost (as in most cases), the generator simply pockets the surplus and no make-whole payment is needed. The outcome is simple and clear: relative to its bid, a generator can be overpaid but will never be underpaid. This is obviously problematic.

The problem is 2-fold. First, the guarantee of revenue adequacy via make-whole payments ostensibly favors the supply side over the demand side, thus violates the "equitable and two-sided" market principle that predicates the uniform price auction format in the first place. Second, if the recovery of the as-bid costs needs a guarantee, then why not directly pay the generators according to their bids, i.e., pay-as-bid? Paying at MCP and then making make-whole payments to match up with the as-bid cost seems artificial. In fact, the mere existence of, or reliance on, the make-whole payments is evidence that the uniform price payment design is flawed.

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In fact, the mere existence of, or reliance on, the make-whole payments is evidence that the uniform price payment design is flawed – the uniform MCP neither clears the market, nor reflects the true cost of electricity. Ramifications are not limited to the unfairness and complexity (of having to use 'make-whole' payments), but also open loopholes for market exploitation. For example, JPMorgan was found to game the California power market by exploiting the market rules. The essence of their lucrative strategy was to request a sky-high commitment fee while offering an extremely low price ($30/MWh) for energy, enough to make the overall cost profile appear competitive so that their units get selected for dispatch. In the end, they are paid for energy at the MCP, which was higher than their bid, and at the same time reap the high commitment fee via make-whole payments. It is reported that JPMorgan amassed $57 million in improper payments over six months in 2010 and 2011.9

III. The Electricity Market Is Not Perfectly Two-Sided

The classic pictorial economic analysis of the price-quantity relationship involves an upward-sloping marginal supply curve and a downward-sloping marginal demand curve. The two curves intersect at the market equilibrium point, which identifies the transacted quantity and the market price, and also maximizes the social welfare. Although the electricity market is designed to look like a two-sided market in which both suppliers and demanders bid, the suppliers and demanders are not at equitable places: On the supply side, it is relatively easy to estimate the marginal cost of generating a megawatt-hour of electricity, since in most cases, the cost is determined by the fuel cost and the unit's efficiency. However, it is difficult for the demand side, especially for the residential and commercial consumers which constitute a significant portion of the total demand, to identify the true marginal value of electricity. Many consumers regard electricity as an essential product and simply consume at whatever
price that is passed to them. In turn, there is no way for their representative wholesale buyers, e.g., load-serving entities (LSE), to come up with bids that reflect the true marginal values of electricity with any accuracy. Therefore, the design of the electricity market as two-sided in the hope of maximizing social welfare is flawed – it does not correspond to reality. Telling evidence of inequitable market participation is that ISOs primarily use the forecasted demand, in lieu of demand bids, as the input to the dispatch algorithms, whereas they use supply bids for the generation side.

Furthermore, even for those bidders who are able to quantify the marginal values, they are not treated comparably to the generators in the market clearing algorithm and the settlement rules. Specifically, the “consuming unit commitment” issue is practically ignored. Although it is not usually perceived this way, a consumer could legitimately have a commitment requirement. For example, a manufacturing plant might need to run its energy-intensive process or unit for a continuous five hours, i.e., minimum run-time requirement, to start the process it might take some preparation costs, i.e., startup cost, and during each hour of the process, extra staff or other fixed costs to facilitate the electricity consumption might be required, analogous to the no-load cost on the generation side.

Such a scenario, although hypothetical, raises an equity question: if the consumers are obliged to pay the generating unit commitment fees to generators, should not the generators reciprocate by paying the consuming unit commitment fees to consumers?

Finally, the energy market over the grid is nothing like the marketplace of an ordinary commodity. A central dispatcher or auctioneer is indispensable, understanding the settlement algorithm is critical in the design of a sound payment rule.

The unit commitment economic dispatch model is a MIP and can be viewed as a two-stage problem: the unit commitment decision is made in the first stage, whereas the dispatch decision is made in the second stage given the commitment decision. The mathematical model is as follows.

\[
\min_{y \in \{0,1\}} s^T y + Q(y)
\]

s.t. \( My \geq b \) (1)

where constraint (1) contains the minimum up- and down-time requirements and other commitment related constraints, and \( Q(y) \) is the optimal value of the second stage economic dispatch problem.

\[
\min_{x,z,\delta} c^T x
\]

s.t. \( Ax + Bz = d \ (\perp p) \) (2)

\( Ex \geq Fy \ (\perp \eta) \) (3)

\( Qx \geq q \ (\perp \mu) \) (4)

\( Gz + H\delta \geq 0 \ (\perp u) \) (5)

The model minimizes the total as-bid production cost. Variable \( y \) is the commitment decision, e.g., \( y_{k,t} = 1 \) if the unit at network node \( k \) is committed in time period \( t \) (for the remainder, subscripts are omitted as they are easily inferred and the model is presented in vector format). Variable \( x \) represents the dispatch, \( z \) the line flow, and \( \delta \) the voltage angle. Constraint (2) is the power balance equation, (3) represents...
the lower and upper bound constraints on the power output \(x\), which take the commitment \(y\) as a parameter, (4) represents the ramping requirements, and (5) encapsulates all the network constraints, such as line flow equations and line thermal limits. Independent of the model, the multiplier of each constraint is listed between the parentheses behind the corresponding constraint. At an optimal solution of the second stage problem, the value of the multiplier \(p\) of the constraint (2) is the MCP.

In a stylized analysis of the existing market rule, the total payment to generators includes the unit commitment cost \(s^T y\) and the energy cost charged at the MCP, i.e., \(d^T p\). It has been recognized that the solution to the above UCED model does not necessarily minimize the total payment \(s^T y + d^T p\), which is the true cost of electricity from the consumers’ point of view. Efforts have been made to minimize the consumer payment by solving a bi-level problem, which could be reformulated as a MIP with big-M constraints. However, no efficient method has been reported to solve this MIP for large-scale instances, due to the inefficiency of the large number of big-M constraints. The bi-level model could also be reformulated as a nonlinear program, which is nonetheless hard to solve due to the non-convex constraints introduced to express the lower-level optimality conditions. Even for toy problems, experiments have shown that the minimum payment solution obtained by this model is usually not a practically desirable solution.

We believe that the root cause of the mismatch between the minimum cost solution and the minimum payment solution lies in the pricing rule. In particular, only pricing the power balance constraint (2) and neglecting the marginal prices of the other constraints lacks justification from many perspectives. A theoretically correct way is to price all the ED constraints (2)–(5) with the corresponding multipliers \(p\), \(\eta\), \(\mu\) and \(u\), respectively. This will yield a total energy payment of

\[
d^T p + (Fy)^T \eta + q^T \mu + 0^T u
\]

which is the dual objective of the second stage LP. By LP duality, the above expression is equal to \(c^T x\) at the optimal solution, hence the total payment becomes

\[
c^T x + s^T y
\]

which is exactly the total as-bid cost that is being minimized in UCED. On the individual level, it prompts a pay-as-bid scheme: pay a generator according to its bid, for both the unit commitment part and the energy part. This pay-as-bid scheme not only eliminates the inconsistency between the minimum-cost solution and the minimum-payment solution, but also induces accurate valuation of generating assets (to be illustrated in the example given below), hence enhances the basis for a truly competitive supply market. Furthermore, the approach is more practical since UCED (minimum cost) is much easier to solve than the bi-level program (minimum payment). The pay-as-bid scheme has been discussed extensively in the literature but not adopted in the U.S. market. In fact, most electricity markets use the uniform-price auction format, and only a few adopt the pay-as-bid format. For example, the electricity market in Britain and Iran switched to a pay-as-bid format in 2001 and 2003, respectively, and Italy has recently decided to follow suit. A similar move was considered in California in 2001 but was not implemented. We list the common reasons against the pay-as-bid scheme (between quotation marks), and our arguments against them.

1. “It distorts the competitive nature of the market by giving no incentive for technological innovation to suppliers, since the pay-as-bid resembles the cost-based pricing of the rate-of-return regulation.”

The competitiveness of the market does not depend on the pricing...
scheme (as long as it is a fair one), but primarily comes from its openness, i.e., the transmission facility is no longer the property of a single generation firm or utility as in the past; instead, any firm willing to and capable of participating in the market can have non-discriminatory grid access. As more and more suppliers enter the market, competitiveness is a natural result. To the contrary of the claim, since the pay-as-bid rule explicitly prices every component of the generators’ operating and cost characteristics, the incentives for technological innovation and efficiency improvement will be more explicit.

2. “It forces the suppliers to depart from bidding their true marginal costs in order to make a profit, whereas it is believed that under the pay-at-MCP scheme, suppliers have every motivation to bid their marginal costs.”¹⁸ First, a sound market design should allow suppliers to behave however they deem best for their interests, within the parameters of the market rules. It is acceptable and perfectly natural to include a profit margin in the bid under the pay-as-bid scheme. A fuel-efficient generator could afford a higher profit margin while being competitive in its bid profile; likewise, an inefficient generator may have little or no profit. This is a sensible and healthy market outcome, and it poses no systematic discrimination toward suppliers in any tier (baseload and peak load, etc.). Second, the claim that “suppliers have every motivation to bid their marginal costs” is a textbook scenario and does not apply to the electricity market on the grid (as we demonstrate below). Numerous studies have been conducted on supplier bidding strategies, and real-world examples of manipulative market behavior abound and are happening, e.g., the JPMorgan manipulative bidding story.

3. “Under the pay-as-bid rule, a supplier’s best offer is a price equal to its best predicted MCP. If all the suppliers were able to predict the MCP with 100 percent accuracy, pay-as-bid would result in the same market outcome as pay-at-MCP; otherwise, prediction inaccuracies would lead to dispatches departing from the ‘merit order’, and consumers would end up bearing the costs of such inefficiency.”¹⁹ If a single price were all that comprises a generator’s offer, then all the unit commitment related issues would be gone and the above claim would be correct. However, the reality is that the generators submit multiple blocks of energy offerings each with a price and an incremental quantity, as well as unit commitment requirements that will affect the dispatch (as we also demonstrate below). Therefore, it is premature to assume the criticality of predicting the MCP (under the pay-as-bid rule, MCP means the highest accepted offer price). In fact, even knowing the MCP with certainty does not enable a generator to find an optimal bid in the sense that its profit will be maximized. For example, if a generator bids aggressively in the unit commitment parameters such as requiring a long minimum-up time, it might lose (the opportunity of being selected), irrespective of its price bid, to a competitor who bids modestly in the same parameter. It is also unrealistic to impose an extreme risk-seeking attitude on all suppliers, i.e., always striving to bid (and get paid) at the highest possible price and disregarding the risk of not being dispatched at all. Furthermore, there is not a clear “merit order” without knowing the commitment, which by itself is a decision variable in the ISO’s market clearing algorithm.

The following stylized example demonstrates the advantage of pay-as-bid in the unit commitment context. For simplicity, let us consider one time period and a generator (GEN1) with one block of price-quantity offer, in particular, its marginal cost is $10/MWh for up to 100 MW, and a startup cost of $200. Suppose that the MCP is...
going to be $15/MWh and will not be affected by GEN1’s offer, be it accepted or not. Further assume that GEN1 knows all this information. How should GEN1 bid?

For the system operator, the net benefit of accepting GEN1 is

\[ \left(15 - p\right) \times 100 - 200 \]

where \( p \) is the offer price of GEN1. Clearly, GEN1 will be accepted only if \( p \leq 13 \) and $13/MWh is the market value of GEN1 (although the MCP is 15). Under pay-as-bid, GEN1 will optimally offer 13 and realize a profit of \( \left(13 - 10\right) \times 100 = 300 \). However, under pay-at-MCP, GEN1 could offer any price lower than 13 to get paid at the MCP of 15, obtaining a profit of \( \left(15 - 10\right) \times 100 = 500 \). This amounts to a $200 over compensation to GEN1, and consumers will bear the cost.

In the same setting, now suppose that GEN1’s marginal cost is 14 instead of 10. Since \( 14 > 13 \), GEN1 is not cost-effective and should not be accepted. Under pay-as-bid, GEN1 can do nothing about it; but under pay-at-MCP, it could still offer any price lower than 13 to get accepted, and finally realize a profit of \( \left(15 - 14\right) \times 100 = 100 \). This would be a loss of efficiency at the consumers’ cost.

For another scenario, if the dispatched quantity of the marginal offer was less than 100 MW and the marginal unit had a higher startup cost, e.g., $400, then GEN1 could potentially replace the marginal unit. In this case, the net benefit of accepting GEN1 will be

\[ \left(15 - p\right) \times 100 - 200 + 400 \]

which indicates that GEN1 could bid up to $17/MWh and still get dispatched. This represents a case where a lower marginal-cost bid (i.e., 15) is not accepted while a higher marginal-cost bid (i.e., 17) is accepted. Clearly, the “merit order,” if one exists, is not simply a ranking of the marginal costs in the bids.

It can be seen from the above examples that in the unit commitment context:

1. The MCP does not represent the market value of all accepted units.
2. Under pay-as-bid, a generator’s best bid is not necessarily its predicted MCP.
3. The market outcome will not be the same under pay-as-bid and pay-at-MCP, even if participants know the MCP with certainty.
4. Under pay-as-bid, it is difficult to fool the system operator, as a generator’s actual payment price is consistent with its bid price.

V. Suppliers’ Response Will Be Rational under Pay-as-Bid

While there is abundant literature on bidding strategy and market equilibrium under various auction designs, the study of bidder’s behavior under pay-as-bid in the UCED context is still limited. We leave a comprehensive discourse on this subject to future work, and outline a general model accompanied by a simulation experiment, simply to make our point.

Let \( O_i := (S_i, A_i, b_i, c_i, F_i, q_i) \) represent the offer (bidding) choice of unit \( i \) and let \( \theta(O_i) \) be the return secured if the offer \( O_i \) is accepted, which is easy to calculate. However, whether or not an offer \( O_i \) will be accepted depends on the circumstances (e.g., the system demand and other suppliers’ offers), and from unit \( i \)’s perspective, is a Bernoulli random variable (1 if accepted and 0 otherwise). The expected value of this random variable is the probability of \( O_i \) being selected, denoted by \( p(O_i) \), which is to be approximated via repeated experiments (market participation). In the long run, as the market is at the state of (dynamic) equilibrium, \( p(O_i) \) can be well approximated. Therefore, for a risk-neutral unit \( i \), its profit maximization problem is

\[ \max_{O_i \in \mathcal{O}} \theta(O_i) \cdot p(O_i) \]  

where \( \mathcal{O}_i \) is the feasible set of unit \( i \)’s offer choices.
Realistically, $\theta(O_i)$ and $p(O_i)$ are inversely proportional, and their product is thus concave-shaped and has a maximum. For expositional purpose, we now consider a simple case where the energy bid price $c_i$ is parameterized by a markup factor and other components of $O_i$ are fixed. Based on this case, we show that (6) has a solution and that pay-as-bid recognizes and rewards bidders’ technological advantage appropriately.

We use the PJM bidding data with masked generator names (publicized by FERC for research purposes). The original data set contains 1,011 generators, from which we selected the first 50 generators for the simulation experiments. Table 1 lists the composition of the selected portfolio of generators.

We first demonstrate the relations between a generator’s profit markup level in the bids and its realized profits. We pick a BIT generator “GEN6” as the subject, the characteristics and original bids of which are listed in Tables 2 and 3. Keeping everything else constant, we multiply GEN6’s energy bid prices by different markup factors, i.e., from 1 to 11 with increments of 0.5, and for each factor setting observe the realized profits in 100 randomized demand scenarios. Assuming that the original bid prices (with markup factor equal to 1) represent the true marginal costs, we can compute the cost once we obtain the dispatch solution. Under the pay-as-bid rule, a generator’s revenue is just the corresponding term in the objective function of the UCED model, and the profit is calculated by the formulae:

$\text{Profit} = \text{Revenue} - \text{Cost}$

The demand scenarios are generated in two steps. First, we arbitrarily create a base demand curve for a 24-hour period using the following formulae,

$$d_t = 10,000 + 3,000 \sin \left( \frac{2\pi t}{24} - 1 \right)$$

where $d_t$ is the base demand in hour $t$. The sine function is used to mimic the demand variations throughout a day. Next, the actual demand in hour $t$ is treated as a uniform random variable distributed in $[d_t - 100, d_t + 100]$, and 100 samples are drawn for each $t$ to form 100 daily demand scenarios.

To illustrate the effects of a unit’s commitment costs on its market competitiveness, we do the same experiments on GEN5, which we make to have the same characteristics and cost parameters as GEN6, except for a much lower no-load and startup cost, one tenth of that of GEN6. Low commitment cost increases the cost-efficiency, hence competitiveness, of a unit, so we expect to see the advantage of such features of GEN5 over GEN6 in the experiments. The results are summarized in Figure 1.

In the figure, for each profit markup level on the horizontal axis, we plot the mean (the dots, which are connected by the curves), the 25 percent and 75 percent percentile (the lower and upper borders of the boxes,
respectively), and the minimum and maximum (bottom and top points of the sticks, respectively) of the realized profit over the 100 demand scenarios. For the boxes that are invisible in the plot, they are actually concentrated (both upper and lower borders) at zero profit level. The mean values represent the expected profits, whereas the candlesticks to some extent indicate the risks. Rich insight can be drawn from the figures.

1. The expected profit is indeed a concave-shaped function of the profit markup in the bid, and possesses a maximum. This validates the bidder’s profit maximization model as discussed earlier. It can also be seen from the candlesticks that the higher the markup level, the riskier the bid will be, in the sense of a higher probability for the bid to be rejected.

2. GEN5 exhibits a more competitive profit curve as expected. GEN5 receives a higher maximum expected profit (at optimal markup 5.5) than that received by GEN6 (at optimal markup 4.5). The difference is $8,206, or 11.6 percent. Besides, GEN5’s profit curve has a wider and flatter top, which implies a broader risk tolerance for over-bidding. For example, GEN5 could tentatively bid at a markup level of 7 and not bear a big opportunity loss compared to the optimal bid, while bidding at 7 would be an immediate disaster to GEN6 as its 75 percent highest profit would be 0.

In Figures 2 and 3, we plot the expected profit (or payoff) of GEN5 and GEN6, respectively, as a function of the markup levels of both GEN5 and GEN6. Such payoff matrices and plots are useful tools to analyze the market equilibrium in both the game theoretical and optimization framework. Although a detailed equilibrium study under the proposed pay-as-bid will be deferred to future work, some qualitative and sensible insight is readily available in the plots.

1. A generator’s expected profit is consistently concave-shaped as a function of its own profit markup level. This can be seen by fixing the competitor’s
markup to any level and observing the resulting slice of the 3D plot. This once again validates the profit maximization model postulated earlier.

2. A generator’s expected profit is also influenced by its competitor’s bid. Take GEN5 (Figure 2) for example, when its competitor GEN6’s markup is in the low range (which means higher competitiveness), GEN5’s maximum profit is relatively low (around $2.5 \times 10^4$), this is because the market is shared with GEN6. As GEN6’s raises its markup level, its competitiveness gradually diminishes and so does its market share, so GEN5 gains more market share and therefore achieves a higher maximum profit (of around $7.5 \times 10^4$). Such effects are completely intuitive, and the same pattern is present in Figure 3, too.

3. The differences between GEN5 and GEN6, including their maximum profit and the risk tolerance for over-bidding, are clearly exhibited in the two figures, and are consistent with the observations from Figure 1.

VI. Conclusions

The existing payment rules, i.e., paying at a uniform MCP for energy and relying on “make-whole” payments to recoup the unit commitment costs, has been shown to be problematic both in theory and in practice. The root cause of the problem is 2-fold: (1) the unavoidable unit commitment constraints make a uniform MCP nonexistent; (2) the supply side and demand side do not behave, and are not treated, equitably in the market. In this context,
pay-as-bid is a viable payment rule to be considered by policymakers. In theory, pay-as-bid is equivalent to a comprehensive marginal pricing scheme, as it prices every component of a unit’s characteristics, fairly and accurately evaluates a unit’s efficiency, and therefore provides clearer signals for innovations and improvements. If implemented, it is expected to generate rational responses from the bidders, and will foster a healthier competition environment.

Endnotes:

1. For example, approximately 45 percent of New York electricity is transacted in the NYISO day-ahead market, 5 percent is transacted in the NYISO real-time market, and half through bilateral contracts. See NYISO Market Training Material: NY Market Orientation Course, 2005.

2. In the remainder of the article we use the acronym “ISO” to indicate an independent system operator or regional transmission organization.

3. In this article, we use the terms “offer” and “bid” interchangeably, although in some literature they refer to “supply offer” and “demand bid,” respectively; we use the word “generator” to indicate a generating unit, a generation plant, or a market participant making a supply offer, depending on the context. A generator’s as-bid cost has three components: energy, no-load, and startup. For the realized cost to the ISO/RTO (or the consumers), no-load and startup costs are determined by the unit commitment decision, and the energy cost is determined by the dispatch decision under a fixed commitment.

4. See Xiaohong Guan, Peter B. Luh and Houzhong Yan, An Optimization-Based Method for Unit Commitment, 

5. The MIP formulation is predicated on the use of linear (also called DC) power flow equations instead of nonlinear (also called AC) power flow equations in modeling the network. Most ISOs use DC power flow equations.


8. Detailed definition and calculation can be found in the respective ISO/RTO’s market rules and tariffs. Since they are scattered in multiple documents, we do not list specific references here.


11. For industrial consumers, electricity is a resource, like steel, wood, and other raw materials, for which the marginal value may be estimated by the shadow price from the production model.

12. Complex forms of demand bids are not present in the ISO/RTO markets, although they have been discussed in the literature, for example, Chua-Liang Su and Daniel Kirschen, Quantifying the Effect of Demand Response on Electricity Markets, IEEE Trans. on Power Sys. 24:3(2009) at 1190–1207.


17. See Peter Cramton and Steven Stoft (2007), Why We Need to Stick with Uniform-Price Auctions in Electricity Markets, ELEC. J., Jan./Feb. 2007, at 26–37.


19. See footnote 18.

20. This happens if the accepted quantity of the marginal offer is greater than GEN1’s capacity of 100 MW. In this case, if GEN1’s offer is later accepted, it amounts to a deduction of 100 MW from the marginal quantity and the marginal price remains unchanged.